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Semantic Incompleteness of Quantum Physics and EPR-Like Paradoxes

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In the approach to quantum physics (QP) forwarded by the author an *a priori* formalization of the observative language of the theory is yielded. It is shown here that this formalization allows one to avoid both ontological realism and verificationism, which are the philosophically opposed positions that are usually assumed in the debate on the paradoxes that seem to follow from the analysis of the Einstein, Podolsky, and Rosen (EPR) thought experiment. Some recent results are summarized (in particular, the semantical incompleteness of QP) obtained by the author in the framework of the aforesaid approach, and it is shown that they can be used in order to deal with some EPR-like paradoxes. Thus one can legitimately affirm that at least some of them can be a consequence of semantical ambiguities and of the acceptance of a philosophical dichotomy which is not logically unavoidable.

1. INTRODUCTION

Every treatment of foundational topics in physics meets from the very beginning with the problem of avoiding the traps and the semantical ambiguities of the natural language, enriched by technical and mathematical symbols, by means of which physical theories are usually stated. Indeed, the different constitutive parts of a language (like syntax, semantics, truth theory, interpretation) are deeply interwoven in natural languages, making it difficult to analyze them rigorously and single out the implicit assumptions in our reasonings. In addition, natural languages often lead to a mingling of different linguistic or metalinguistic levels, which is an inexhaustible source of paradoxes.

These reasons have led me to think that some foundational problems can be greatly enlightened if the language of physics (to be precise, suitable

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fragments of the language of physics) is formalized by means of logical, epistemological, and linguistic tools. Of course, such a formalization cannot be neutral, since it depends on epistemological and philosophical choices; but it makes these choices explicit and comparable, and assures a high degree of self-consistency.

I have proposed a first formalization of a part of the language of quantum physics (QP) in some papers (see, in particular, Garola, 1991) where I tried to answer the old question of whether nonstandard logics are actually needed in QP. My starting point was the conviction that quantum logic (QL) cannot provide an adequate solution to this problem, for various reasons that I have discussed elsewhere (Garola, 1992*b*,*c*); among these is the basic fact that some laws of QP are usually presupposed (sometimes implicitly) when defining the properties of the connectives in QP. Therefore, I have made an attempt of answering the question in a context where a suitable formalized language L is constructed *a priori* with respect to the laws of QP (L is an *observative* sublanguage of the formalized language L^* that one ought to construct in order to formalize completely the language of QP). But the arguments mentioned above show that such a formalization can be considered relevant because of more general reasons, and that its consequences can be far-reaching. I would like to deal in this paper with some of these consequences.

With this aim in mind, I will first show that my approach avoids both ontological realism and verificationism, which are deeply involved in the problems of the completeness of QP and with EPR-like paradoxes (Section 2). This leads to some nontrivial results which have already been discussed elsewhere (Garola, 1992*a*), like the semantical incompleteness of QP and a sharp semantical distinction between pure states and their characterizing physical properties (Section 3). Furthermore, some new results can be obtained in this framework regarding EPR-like paradoxes, and I will anticipate them briefly in the final part of this paper (Section 4). I will also comment on the fact that my approach recovers some traditional positions in QP, which are thus founded on a more rigorous and explicit semantical basis (Section 5).

Since the treatment will continuously make reference to the language L, I will begin my discussion with a brief recapitulation of the main features of this formalized language in the next section.

2. THE OBSERVATIVE LANGUAGE L

Following the program outlined in the Introduction, let us consider the language L which formalizes an observative part of the whole language of QP. Then L is a predicate calculus of the first order with monadic predicates only. The *alphabet* of L contains the following sets of signs:

(i) A set X of *individual variables* $x, y, \ldots$.

(ii) A set $\mathscr{P}$ of *monadic predicates*, partitioned in two subsets, the set $\mathscr{S}$ of *symbols of states* $S, S_1, S_2, \ldots$, and the set $\mathscr{E}$ of *symbols of effects* $E, E_1, E_2, \ldots$.

(iii) The set of standard connectives $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$.

(iv) The set of standard quantifiers $\{\ni, \forall\}$.

(v) A family $(\pi_\Delta)_{\Delta \in \mathscr{B}(\mathbf{R})}$ of *statistical quantifiers*, parametrized by the ring $\mathscr{B}(\mathbf{R})$ of all Borel sets on the real line.

(vi) The set of *auxiliary signs* $\{(\cdot), /\}$

By adopting the Ludwig (1983) analysis of experimental apparatuses, with some differences that are important but that will not be recalled here for brevity's sake, every symbol of state S is interpreted, intensionally, as a class $[p_S]$ of physically equivalent preparing devices, or *state*; analogously, every symbol of effect E is interpreted, intensionally, as a class $[e_E]$ of physically equivalent dichotomic (yes–no) registering devices, or *effect* (by abuse of language we will not distinguish here between *symbol of state* and *state*, *symbol of effect* and *effect*). The extensions of these predicates are defined by making reference to the concept of *laboratory*, which is a space-time domain in the actual world; we denote by I the set of all laboratories, and by $\tilde{I}$ a suitable subset of laboratories which are *statistically relevant*, i.e., intuitively, that contain a large number of individual physical systems, or *physical objects*. Thus, for every laboratory i, the extension $\rho_i(S)$ of the state S is a set which is physically interpreted as the set of all physical objects prepared in i by devices in $[p_S]$; similarly, the extension $\rho_i(E)$ of the effect E is a set which is physically interpreted as the set of all physical objects in i which would yield the answer yes if tested by means of any device in $[e_E]$. Consequently, a *domain* D_i is associated with every laboratory i, which is interpreted as the set of all physical objects that are prepared in i by means of preparing devices; furthermore, an *interpretation* σ of the (individual) variables of L is defined as a mapping

$$\sigma: \quad (i, x) \in I \times X \rightarrow \sigma_i(x) \in D_i$$

which, for every laboratory i maps the variables in L into elements of D_i (by abuse of language we call *physical objects* the variables themselves whenever an interpretation σ is implied).

Consistently with the above interpretation of L it can be assumed that, for every laboratory i, the set of all extensions of states is a partition of D_i (which is an important difference between this approach and Ludwig's). On the contrary, the extensions of different effects can have nonempty intersections, and the set $\mathscr{E}$ can be partially ordered by the relation $<$ defined as follows:

$$\text{for every } E, E' \in \mathscr{E}, \quad E < E' \quad \text{iff} \quad \text{for every } i \in \tilde{I}, \quad \rho_i(E) \subseteq \rho_i(E').$$

The poset $(\mathscr{E}, <)$ contains a proper subposet $(\mathscr{E}_E, <)$, the poset of all *symbols of exact effects* (briefly, *exact effects*), whose elements denote equivalence classes of idealized dichotomic registering devices which exactly test whether the value of a given physical observable belongs to a given Borel subset of the real line. Then it can be proved under suitable physical assumptions that $(\mathscr{E}_E, <)$ is a lattice and is isomorphic to the Mackey (1963) lattice of questions, or the Piron (1976) lattice of propositions; hence it is a complete, orthocomplemented, atomic lattice, which is distributive in classical physics (CP), weakly modular, and satisfying the covering law in QP. It follows that all elements in this poset can be associated with (testable) physical properties, and by abuse of language we simply call them *properties*.

Let us come to the concept of truth in L. The atomic well-formed formula (wff) $S(x)$ [respectively, $E(x)$] will be said to be true in the laboratory i for a given interpretation σ iff $\sigma_i(x) \in \rho_i(S)$ [respectively, iff $\sigma_i(x) \in \rho_i(E)$]. Thanks to the above interpretation of L, it follows that $S(x)$ is true in i iff the physical object $\sigma_i(x)$ has been prepared in i by means of some preparation procedure characterizing S [hence, for every state S the truth value of the wff $S(x)$ is known whenever the interpretation σ is given]. Whenever $S(x)$ is true in i we say that *x is in the state S in laboratory i*, leaving the reference to the interpretation σ implicit. Analogously, $E(x)$ is true in i iff the physical object $\sigma_i(x)$ would yield the answer yes if it should be tested by means of a registering device characterizing E [hence for every effect E the truth value of the wff $E(x)$ can be considered assigned but not necessarily known whenever the interpretation σ is given]. Whenever E is an exact effect and $E(x)$ is true in the laboratory i we say that *x has the property E in i*, or that *E is true in i for the physical object x*, again leaving the reference to the interpretation σ implicit. Finally, a truth value is assigned to all molecular and quantified wffs of L using standard definitions in classical logic (CL), suitably extended so that truth values can be attributed to wffs containing statistical quantifiers.

Let me discuss briefly the peculiarities of L that interest us here. First, note that all predicates are defined *operationally* in terms of preparations and registering devices. This meets orthodox operational requirements in QP and makes it possible to avoid any kind of ontological realism, which implies a flexible philosophical attitude, since theoretical terms can be conceived as constructed in order to explain (objective) empirical facts and can therefore be changed, when needed, without being bounded by realistic preconceptions on their actual existence (whatever this means). Second, notice that the truth theory embodied in the semantics of L is classical (Tarskian); thus the logic of a physical theory stated by means of L does not depend on the theory itself, which establishes rationality criteria that hold *a priori* with respect to the theory, and no use is made of the verificationist truth theory, that is, no identification occurs between truth and the knowledge of truth, or *epistemic accessibility* (I have recently observed in some of the papers quoted above that verificationist theories can be recovered in our context as theories of the metalinguistic concept of *testability*).

3. STATES, PROPERTIES, AND SEMANTICAL INCOMPLETENESS OF QP

The remarks on L at the end of Section 2 are particularly interesting for the objective of the present paper. Indeed the old problems of the incompleteness of QP and of the paradoxes that would come out in QP whenever a careful analysis is made of the thought experiment presented by Einstein, Podolsky and Rosen (EPR) in their overquoted paper (Einstein *et al.*, 1935) are closely linked with the problems of physical realism and the notion of truth that should be adopted in physics.

I do not intend to discuss now these topics from a philosophical viewpoint. I will only point out that some paradoxes which have been charged to QP have been ascribed to the adoption (implicit or not) of a realistic attitude. On the other hand, there are some weaknesses in the answers of orthodox quantum physicists to the arguments of their "realistic" opponents that follow in my opinion from the adoption (implicit or not) of a verificationist theory of truth and meaning. Thus, it is important to observe that the attachment to realism or, alternatively, to verificationism is often based on philosophical prejudices. In particular, there are epistemological and logical arguments for considering it untrue that a realistic attitude is necessary in order to warrant the objectivity (intersubjectivity) of physics. Analogously, one can seriously question the Dummett (1975) opinion that the adoption of a correspondentistic theory of truth necessarily implies the acceptance of a metaphysical realistic attitude. On the other hand, I think that the criticisms by Russell (1940), Carnap (1949, 1966), and Popper (1969) of the verificationist theories of truth (more precisely, of the identification in these theories of the semantic notion of truth with the pragmatic criterion of truth) are sound, and I do not think that the adoption of a theory of this kind is needed in order to prevent the introduction of "metaphysical" assumptions in physics.

The above remarks suggest that an interpretation of QP which avoids both ontological realism and verificationism, together with the related problems, yet keeping a Tarskian theory of truth (which we consider a rigorous explication of the classical correspondentistic theory of truth), may provide a suitable background for discussing the completeness of QP and EPR-like paradoxes. Now, the language L just embodies these features, as we have seen at the end of Section 2. Thus it is not surprising that an inquiry on the completeness problem by using L may lead to some interesting results (Garola, 1992*a*). Let me briefly summarize them.

(i) QP is *semantically incomplete* with respect to the wffs of the form $E(x)$, with E an exact effect, in the sense that the knowledge of the truth value of a wff of the form $S(x)$, with S a state, in a laboratory i never allows one to deduce the truth value in i of *all* wffs of the form $E(x)$ by making use of the laws of QP (even if S is a *pure* state; the meaning of the word *pure* in our context will be clarified in the following).

The above intuitive result can be expressed more precisely by introducing, for every physical object x and laboratory i, the set $\mathscr{E}^{\mathrm{T}}_{ix}$ of all true properties of x in i (it must be recalled that the interpretation σ is implied), and for every state S, the set $\mathscr{E}_S$ of all properties that are true for every laboratory $i \in \tilde{I}$ and for every physical object in the state S (*certainly true domain of* S). Then, trivially, $\mathscr{E}_S \subseteq \mathscr{E}^{\mathrm{T}}_{ix}$ if x is in the state S. One can show that $\mathscr{E}_S$ is identifiable with the set of all properties that can be *predicted* to be true in the laboratory i for a physical object x by making use of physical laws and of the assumption that x is in the state S in i. Now, it can be proved that $\mathscr{E}_S \subset \mathscr{E}^{\mathrm{T}}_{ix}$ ($\subset$ denotes strict inclusion here) in QP; this makes the above statement about the incompleteness of QP more precise (of course, one would obtain analogous results by taking into account the set $\mathscr{E}^{\mathrm{F}}_{ix}$ of all false properties of x in i).

The above strict inclusion leads to some important consequences. First, it implies that a change of state of a physical object does not necessarily modify its physical properties in QP (while it does in CP), though it modifies the set of properties that are known to be true. Second, it implies that different objects in the same state S can be thought of as endowed with different properties, though the properties in $\mathscr{E}_S$ must be true for them all; this feature, which may seem obvious in QP, is unacceptable for physicists adopting a verificationist theory of truth, who would classify as "nonsensical" the attribution to a given object x of a property E that does not belong to $\mathscr{E}_S$.

(ii) The set $\mathscr{E}_S$ is partially ordered by the restriction to $\mathscr{E}_S$ itself of the order $<$ defined on $\mathscr{E}_E$. It can be proved that it has a minimum E_S, the *support* of S, and that E_S is such that, for every $i \in \tilde{I}$, $\rho_i(S) \subset \rho_i(E_S)$ in QP. Thus the mapping $g\colon S \in \mathscr{S} \to E_S \in \mathscr{E}_E$ associates a property to every state.

The bijectivity domain $\mathscr{S}_P \subseteq \mathscr{S}$ of g will be called *the set of pure states* (indeed it corresponds to the set of pure states in the standard approach to QP) so that, for every pure state S, E_S characterizes S. Nevertheless, pure states cannot be identified with their supports from a semantical viewpoint in QP, since the strict inclusion $\rho_i(S) \subset \rho_i(E_S)$ shows that S and E_S have different extensions, so that the physical object x could have the property E_S even if it cannot be said that "x is in the state S."

It is also important to note that the standard conception of states as "amounts of information" in QP can be considered in the present context as an informal way of interpreting any pure state S on the set $\mathscr{E}_S$ of all physical properties of x that can be predicted to be true whenever x is in the state S (equivalently, S can be interpreted on E_S, as in the Piron approach, since E_S characterizes $\mathscr{E}_S$). Now S has been interpreted in Section 2 as an equivalence class of preparations. Since $\mathscr{E}_S$ characterizes S, the new interpretation is legitimate and I will presently make use of it; but one must be careful to avoid any semantical identification between S and E_S, as we have seen above.

(iii) A binary relation C can be introduced on the set $\mathscr{S}$ which defines the *logical compatibility of states*. To be precise, for every pair of states S and S' we put

$$S\,C\,S' \quad \text{iff} \quad \mathscr{E}_S \cap \mathscr{E}_{S'}^{\perp} = \varnothing = \mathscr{E}_{S'} \cap \mathscr{E}_S^{\perp}$$

(here $\mathscr{E}_S^{\perp}$ denotes the set of all *certainly false* properties of S and $\varnothing$ denotes the empty set), hence $S\,C\,S'$ iff no contradiction occurs between the information embodied in S and the information embodied in S'. Then, C turns out to be an *accessibility* relation (it is reflexive and symmetric but not, generally, transitive). Furthermore, one easily gets the result that, whenever S and S' are pure states, S is logically compatible with S' in QP iff the vectors $|\psi\rangle$ and $|\psi'\rangle$ that represent S and S', respectively, are not orthogonal in the standard Hilbert space model for QP. Hence one can write

$$S\,C\,S' \quad \text{iff} \quad \langle\psi|\psi'\rangle \neq 0$$

(it is interesting to observe that $S\,C\,S'$ iff $S = S'$ in CP).

(iv) Let us consider an idealized quantum measurement, in a laboratory i, of an observable A on a physical object x in a pure state S, let a_j be its outcome, and let us assume that this result is not certainly true in the state S. Then the property

$$E = \text{the observable } A \text{ takes the value } a_j$$

does not belong to $\mathscr{E}_S$, but the measurement itself shows that $E \in \mathscr{E}_{ix}^{\mathrm{T}}$, which could not have been predicted before the measurement because of the incompleteness of QP. This has nothing to do with the properties of x after the measurement, or, more generally, with the transformation of the state of x during the measurement process. However, the latter can be obtained, at least in the case of ideal measurements, by using the projection postulate [which can be regarded as a theorem FAPP, i.e., valid *for all practical purposes*, in some suitable *no-collapse theory* (e.g., Gottfried, 1991)]. Then the state after the measurement is a pure state S_j and the property E can be attributed to x even after the measurement. In the present context it can easily be shown that S_j is compatible with S, so that no inconsistency exists between the information in S and in S_j. Thus it can occur that $\mathscr{E}_S \neq \mathscr{E}_{S_j}$, but all properties in $\mathscr{E}_S \cup \mathscr{E}_{S_j}$ can be simultaneously true for the physical object x both before and after the measurement, though it is impossible to know in QP whether such a situation occurs.

More generally, one can say that the set $\mathscr{E}^{\mathrm{T}}_{ix}$ can remain unchanged during an ideal measurement process even if the state of x changes.

4. EPR-LIKE PARADOXES

The results discussed in Section 3, together with the formalization of the observative language of QP, can greatly help in my opinion in the analysis of EPR-like paradoxes. I would like to summarize in this section some further results recently obtained by Prof. Solombrino and myself on this subject (we will discuss them extensively in a forthcoming paper).

(i) Let us consider the well-known EPR paradox according to Furry and Bohm (e.g., Bohm, 1951). Without entering in technical details on the EPR thought experiment, assuming them to be known, one can briefly say that the paradox consists in considering a compound system x of two subsystems, say 1 and 2, in a pure state, performing an ideal measurement of an observable A_1 on 1, showing that the system is described by a mixture of pure states after the measurement, and exhibiting an argumentation that ought to show that the system is described by the same mixture even before the measurement, so that one obtains two different incompatible descriptions.

The aforesaid reasoning turns out to be incorrect if one takes into account the results in Section 3. Indeed, it is essentially based on the remark that the measurement on 1 does not interact with 2, and the attribution of the mixed state to the system even before the measurement is made as a consequence of the attribution to subsystem 2 of a property of the form

the observable A_2 takes the value a_2 on 2 standard

even before the measurement. But such an attribution (which is correct) does not allow one to determine the state of the whole system before the measurement, as we have seen in Section 3(ii); thus one is not allowed to describe x by means of a mixture before the measurement.

The above result seems to indicate that this kind of paradox disappears in our framework. It is interesting to observe that the solution of the paradox mainly stands on an accurate distinction between two kinds of predicates (i.e., states and effects) in the language of QP.

(ii) Another argumentation that should prove that the quantum treatment of the EPR thought experiment leads to paradoxes (e.g., Bohm and Aharonov, 1957) is based on the remark that a physicist could choose to perform an ideal measurement of an observable B_1 not compatible with A_1 on particle 1. Then the states, say S_{2k} and S_{2s}, of subsystem 2 after the measurement of A_1 and B_1, respectively, are different, though the measurements of A_1 and B_1 do not act directly on 2. Thus particle 2 is said to have different properties in the two cases, which entails that the properties of 2 are determined by the measurement which is performed on 1 without interacting with 2. Hence we should be compelled either to introduce subjectivity in physics (through the arbitrary choice of the observer in 1) or to introduce exotic explanations, like superluminal connections and so on.

Even this argument can be invalidated by making use of the results in Sections 2 and 3. Indeed, it is possible to prove in our context that two cases can occur. First, S_{2k} can be compatible with S_{2s}, so that there is no contradiction between the information embodied in these states. Second, S_{2k} can be incompatible with S_{2s}, but the measurement of B_1 could never lead in this case to a result implying that 2 is in the state S_{2s} whenever a measurement of A_1 would lead to a result implying that 2 is in the state S_{2k}. Thus no inconsistency occurs.

One could look into the above solution a little deeper. Then one sees again that the kind of paradox considered here occurs whenever the extensions of states are identified with the extensions of their characterizing properties. If, on the contrary, states are interpreted as expressing objective information about physical objects, then a change of state (from S_{2k} to S_{2s}) changes the set of properties that are known to be true, but it is incorrect to assume that it necessarily changes the set $\mathscr{E}_{i2}^{\mathrm{T}}$ of all properties of a single sample of subsystem 2 which are true in the laboratory i where the experiment is made. In particular, whenever the states S_{2k} and S_{2s} are compatible, the measurements of A_1 and B_1 provide different but noncontradictory information on 2.

(iii) A third feature of the EPR experiment that leads to paradoxes according to some authors (e.g, Selleri, 1988) is, loosely speaking, the difference in the probabilities at 2 whenever different measurement processes occur at 1 which do not interfere with 2. I will not discuss this paradox in detail here and limit myself to anticipating two remarks and a conclusion.

First, note that every probabilistic statement in physics is a theoretical statement, and that its observative content necessarily refers to frequency measurements. Therefore the basic epistemological requisite that only *operational* predicates appear in the alphabet of L (Section 2) prohibits the introduction of first-order predicates, hence atomic formulas, which can be interpreted as the attribution of a probability value to a physical object.

Second, observe that an abstract notion of probability is not formalized in L, where only statistical quantifiers (hence frequencies and conditional frequencies) appear. Then the typical wff which expresses a physical prediction takes the form $A_r = (\pi_r x)(E(x)/S(x))$, which is interpreted as follows:

"The physical objects in the state S have the property E with frequency r."

It is apparent that the role of *quantifier* attributed to π_r in L prohibits any molecular wff which can be interpreted as the attribution of a probability value to a physical object.

It follows from the above remarks that only statistical statements referring to ensembles of physical objects can be suitably formalized by means of L. Now, it can be shown that the EPR-like paradox that we are considering vanishes whenever only ensembles are considered and the perspective presented in this paper is adopted. Thus L establishes some syntactical and semantical limits which seem to prohibit the formulation of probabilistic paradoxes at the observative level formalized by L itself.

CONCLUSIONS

I would like to close with three remarks on some features of my approach to QP which are suggested by the results discussed here.

First, my analysis of the (observative) language of QP moves along lines that are orthodox in the sense of Gottfried (1991). Indeed, it embodies a standard statistical interpretation of QP; it does not accept Bohr's relational conception of state, and does without verificationism.

Second, I have interpreted physical states as equivalence classes of preparations (as in the Ludwig approach, though the extensions of states are endowed with a different mathematical structure, not with a structure type *selection procedure*, as in Ludwig); but the semantical analysis shows that the canonical alternative interpretation of states as *amounts of information* does not seem trivial nor superficial, as maintained by some authors (e.g., d'Espagnat, 1976).

Third, the semantical incompleteness of QP discussed in Section 3 might constitute a suitable epistemological basis for justifying the attempts of constructing more complete theories. However, the interpretation of QP forwarded here [where different individuals in the same state can be endowed with different properties; note that this feature also appears in other approaches to QP but in a different framework (e.g., van Fraassen, 1981; Dieks, 1989)] does not imply a modification of the theory. It is interesting to observe that the approach by Ghirardi and Rimini discussed in this conference (where all individuals in the same state share the same properties) postulates a modification of the evolution equation of QP.

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